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A Bayesian Hierarchical Multifidelity Model for High-fidelity Predictions of Turbulent Flows

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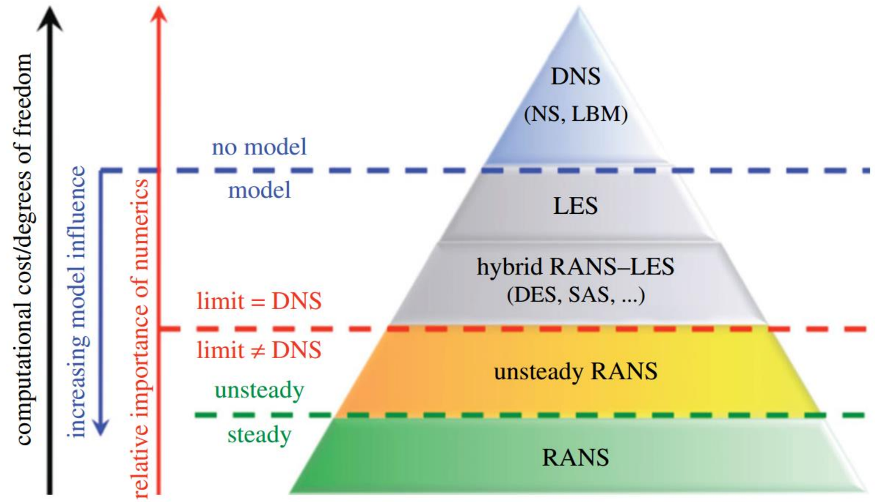
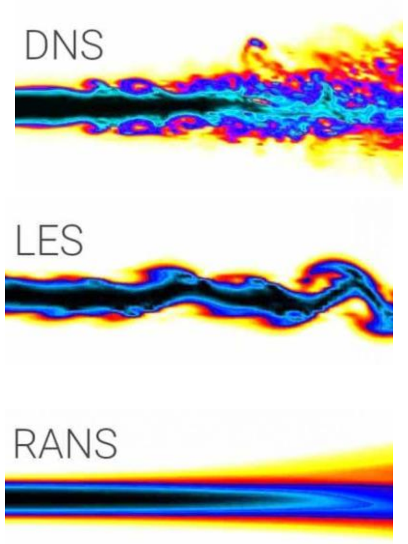
Introduction

Source: https://www.shutterstock.com/

Hierarchy of Turbulence Simulation Approaches



Taken from Sagaut et al. 2013



Source: https://www.idealsimulations.com/resources/turbulence-models-in-cfd/

- Several simulations of turbulent flows are required for outer-loop problems
- **Multifidelity Models (MFM)**: achieve high accuracy given a limited computational budget
- We need a MFM that:
  - is consistent with turbulence modeling hierarchy,
  - can handle uncertainties.

Kennedy-O'Hagan, 2001 (KoH-2001):

$$\begin{cases} y_i = \hat{f}(\mathbf{x}_i, \boldsymbol{\theta}) + \hat{\delta}(\mathbf{x}_i) + \varepsilon_i & , i = 1, 2, \cdots, n_1 \\ y_i = \hat{f}(\mathbf{x}_i^*, \mathbf{t}_i^*) & , i = 1 + n_1, 2 + n_1, \cdots, n_2 + n_1 \end{cases}$$

- $\mathbf{x}$: Controllable inputs (including the design parameters)
- $\mathbf{t}$: Tuning parameters of different models
- $\boldsymbol{\theta}$: Calibrated $\mathbf{t}$

Challenges:

- Computational cost
- Uncertainties

Hierarchical MFM

- Goh et al. 2013 extended the KoH-2001 model to an arbitrary number of fidelities:

- A sequence of fidelities: $M_1 > M_2 > \cdots > M_d$
- Fidelity $\propto$ Cost $\Rightarrow n_1 < n_2 < \cdots < n_d$

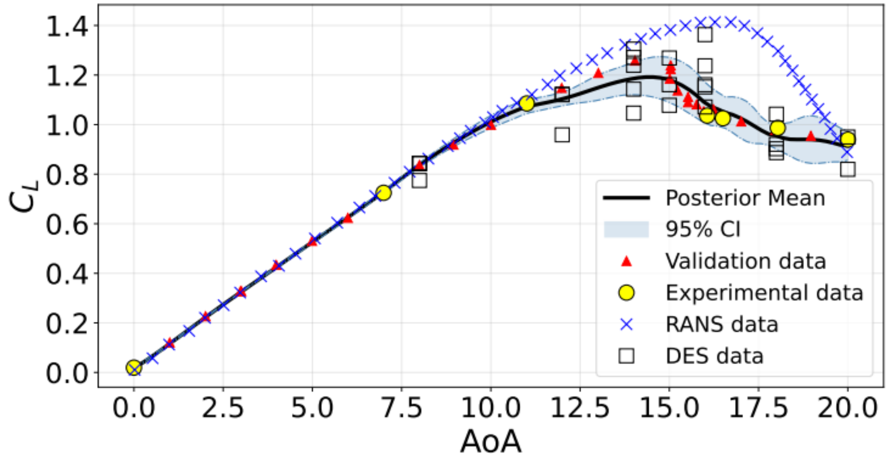
$$\begin{cases} y_{M_1}(\mathbf{x}_i) = \hat{f}(\mathbf{x}_i, \boldsymbol{\theta}_3, \boldsymbol{\theta}_s) + \hat{g}(\mathbf{x}_i, \boldsymbol{\theta}_2, \boldsymbol{\theta}_s) + \hat{\delta}(\mathbf{x}_i) + \varepsilon_{1_i} & , \quad i = 1, 2, \cdots, n_1 \\ y_{M_2}(\mathbf{x}_i) = \hat{f}(\mathbf{x}_i, \boldsymbol{\theta}_3, \mathbf{t}_{s_i}) + \hat{g}(x_i, \mathbf{t}_{2_i}, \mathbf{t}_{s_i}) + \varepsilon_{2_i} & , \quad i = 1 + n_1, \cdots, n_2 + n_1 \\ y_{M_3}(\mathbf{x}_i) = \hat{f}(\mathbf{x}_i, \mathbf{t}_{3_i}, \mathbf{t}_{s_i}) + \varepsilon_{3_i} & , \quad i = 1 + n_2 + n_1, \cdots, n_3 + n_2 + n_1 \end{cases}$$

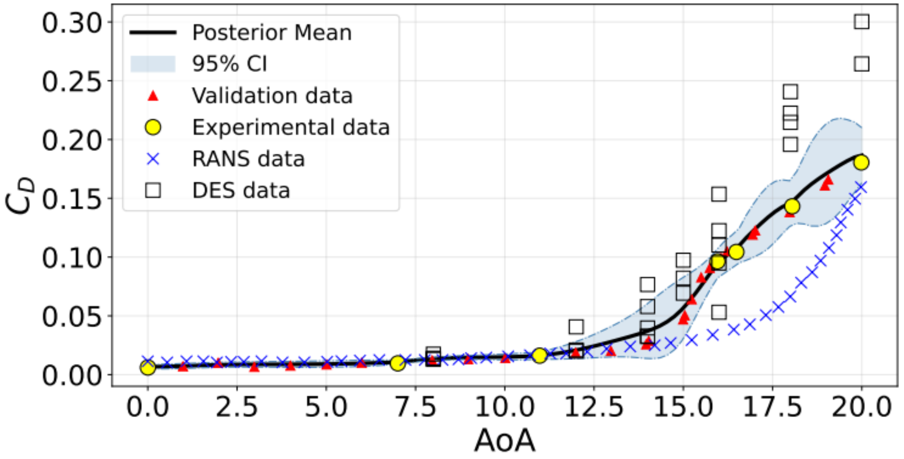
- $\hat{f}(\cdot)$, $\hat{g}(\cdot)$, $\hat{\delta}(\cdot)$: Gaussian processes.
- $\mathbf{t}_k$ and $\boldsymbol{\theta}_k$: Tuning and calibrated parameters of M_k.
- $\mathbf{t}_s$ and $\boldsymbol{\theta}_s$: Tuning and calibrated parameters common between of M_2 and M_3.
- $\varepsilon_{m,i} \sim \mathcal{N}(0, \sigma^2_{m,i})$.
- Apply Bayesian inference with MCMC method (implementation in `PyMC3`).

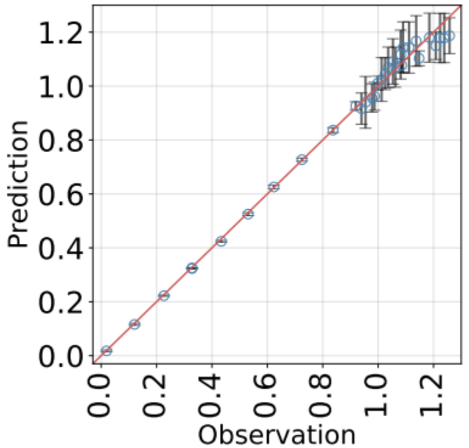
Bayesian Inference: $p(\boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\beta}_\varepsilon, \boldsymbol{\mu} | \mathbf{Z}) \propto L(\mathbf{Z} | \boldsymbol{\theta}, \boldsymbol{\beta}, \boldsymbol{\beta}_\varepsilon, \boldsymbol{\mu})\, p_0(\boldsymbol{\theta})\, p_0(\boldsymbol{\beta})\, p_0(\boldsymbol{\beta}_\varepsilon)\, p_0(\boldsymbol{\mu})$

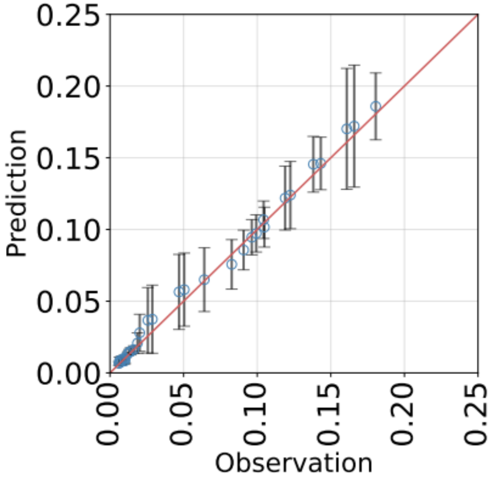
Impact of Angle of Attack on Lift and Drag

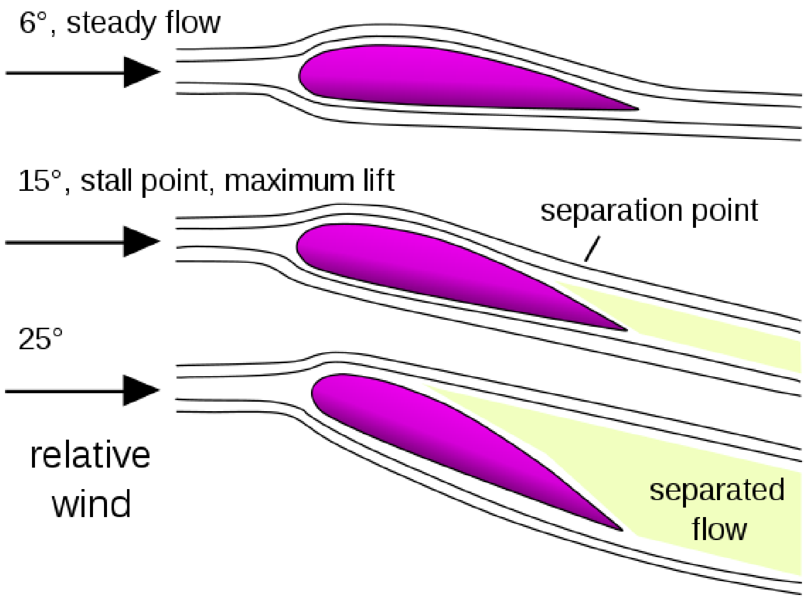
Stall







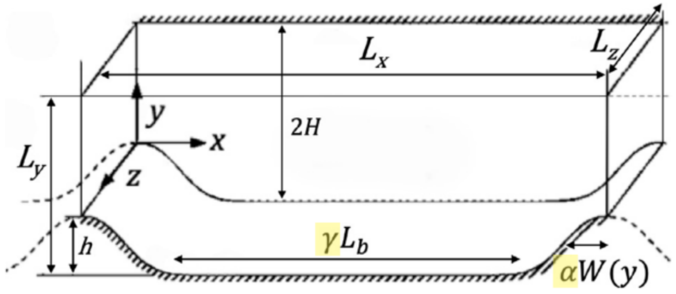




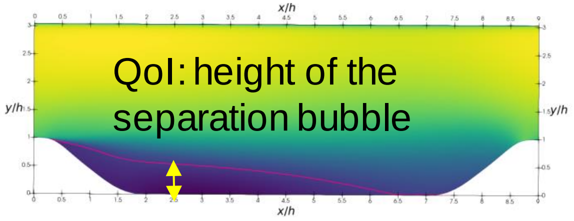
https://en.wikipedia.org/wiki/Stall_(fluid_dynamics))

Fidelities:
- Experiments
- DES (Detached Eddy Simulation)
- RANS

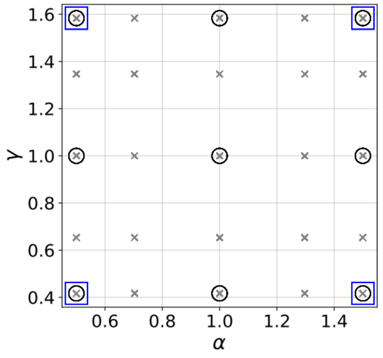
Geometrical Uncertainties



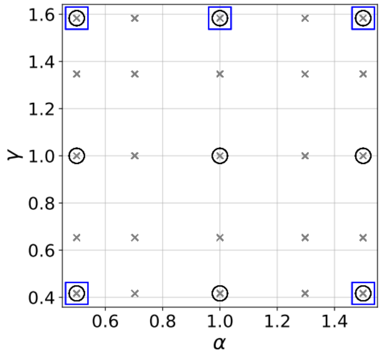
From Voet et al., 2021

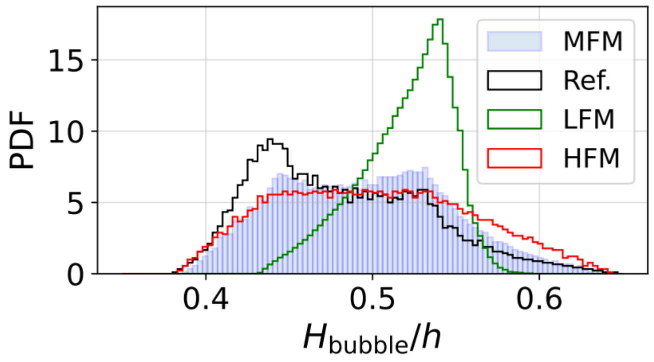


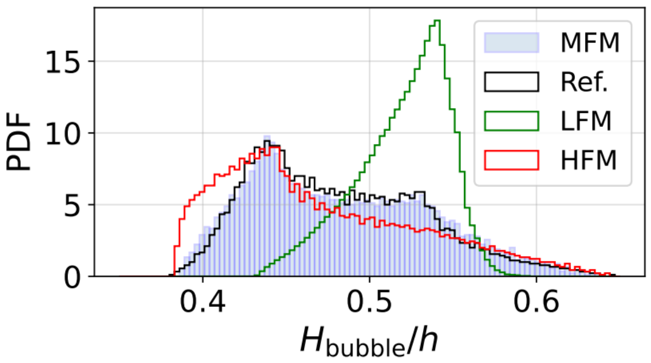
4 DNS
125 RANS



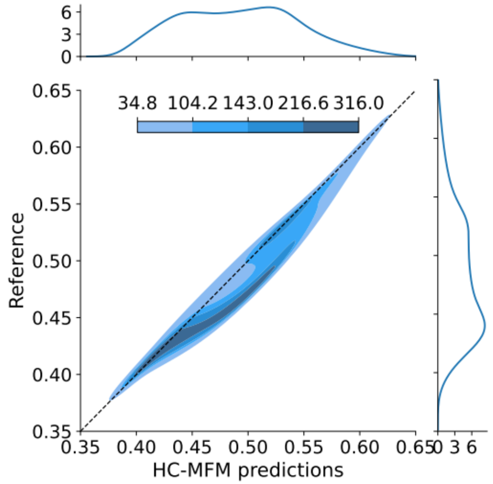
5 DNS
125 RANS

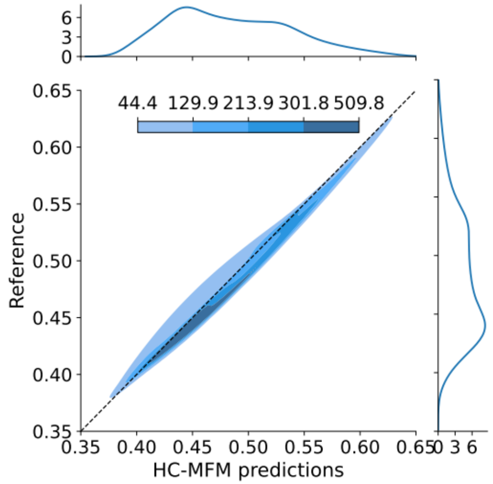


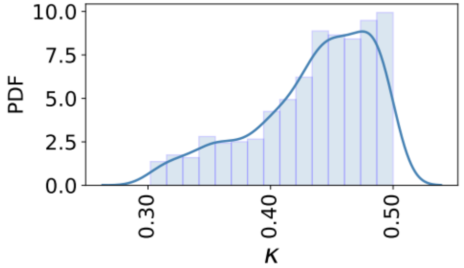


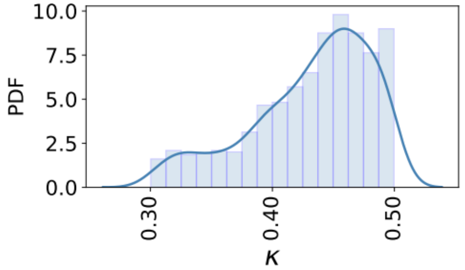


Ground truth: 9 DNS (from Xiao et al. 2020)









RANS modeling parameter

Further reading: S. R., T. Mukha, P. Schlatter, 2022, https://doi.org/10.48550/arXiv.2210.14790

Bibliography

- P. Sagaut, S. Deck, and M. Terracol., Multiscale and Multiresolution Approaches in Turbulence: LES, DES and Hybrid RANS/LES Methods : Applications and Guidelines, Imperial College Press, 2013.
- J. Goh, D. Bingham, J. P. Holloway, M. J. Grosskopf, C. C. Kuranz, and E. Rutter, Prediction and computer model calibration using outputs from multifidelity simulators, Technometrics, 55(4):501–512, 2013.
- M. C. Kennedy and A. O'Hagan, Bayesian calibration of computer models, Journal of the Royal Statistical Society: Series B (Statistical Methodology), 63(3):425–464, 2001.
- J. Salvatier, T. V. Wiecki, and C. Fonnesbeck. Probabilistic programming in Python using PyMC3. PeerJ Computer Science, 2:e55, 2016.
- L. J. Voet, R. Ahlfeld, A. Gaymann, S. Laizet, and F. Montomoli, A hybrid approach combining dns and rans simulations to quantify uncertainties in turbulence modelling, Applied Mathematical Modelling, 89:885–906, 2021.
- F. Bertagnolio, NACA0015 Measurements in LM Wind Tunnel and Turbulence Generated Noise, Number 1657(EN) in Denmark. Forskningscenter Risoe. Risoe-R. Danmarks Tekniske Universitet, Risø Nationallaboratoriet for Bæredygtig Energi, 2008.
- L. Gilling, N. Sorensen, and L. Davidson, Detached eddy simulations of an airfoil in turbulent inflow, 47th AIAA Aerospace Sciences Meeting Including The New Horizons Forum and Aerospace Exposition, 24(AIAA 2009-270):1–13, 2009.
- H. Xiao, J.-L. Wu, S. Laizet, and L. Duan, Flows over periodic hills of parameterized geometries: A dataset for data-driven turbulence modeling from direct simulations, Computers & Fluids, 200:104431, 2020.